OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2604**

Pure Mathematics 4

Tuesday **7 JUNE 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 A curve has equation $y = \dfrac{x(3x-14)}{x-2}$.

(i) Write down the equation of the asymptote parallel to the y-axis, and find the equation of the oblique asymptote. [4]

(ii) Find $\dfrac{dy}{dx}$. Show that the gradient of the curve is always positive. [4]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{x(3x-14)}{x-2} < 20$. [6]

(v) On a separate diagram, sketch the curve with equation $y^2 = \dfrac{x(3x-14)}{x-2}$. [3]

2 **(a)** Find $\displaystyle\sum_{r=1}^{n} r^2(5r+9)$, giving your answer as a product of linear factors. [5]

(b) Show that $\dfrac{3^{r+1}}{r+1} - \dfrac{3^r}{r} = \dfrac{3^r(2r-1)}{r(r+1)}$, and hence find the sum of the first n terms of the series

$$\frac{3 \times 1}{1 \times 2} + \frac{3^2 \times 3}{2 \times 3} + \frac{3^3 \times 5}{3 \times 4} + \frac{3^4 \times 7}{4 \times 5} + \dots.$$ [6]

(c) Prove by induction that $\displaystyle\sum_{r=1}^{n} \frac{2r^2-1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$. [9]

3 In this question, give all answers in an exact form, with arguments in radians between $-\pi$ and π.

(a) Two complex numbers are $\alpha = \sqrt{2}(\cos\frac{1}{12}\pi + \mathrm{j}\sin\frac{1}{12}\pi)$ and $\beta = -4 + 4\mathrm{j}$.

(i) Find the modulus and argument of each of α, β and $\frac{\beta}{\alpha}$. Illustrate these three numbers on an Argand diagram. [8]

(ii) Express $\frac{\beta}{\alpha}$ in the form $a + b\mathrm{j}$. [2]

(iii) On your Argand diagram, indicate a length which is equal to $|\alpha - \beta|$, and find the exact value of $|\alpha - \beta|$. [5]

(b) Find the complex number z which satisfies $\mathrm{j}z + (2 + \mathrm{j})z^* = 10 - 2\mathrm{j}$. [5]

4 **(a)** Four points have coordinates A(3, 4, 7), B(13, 9, 2), C(1, 2, 3) and D(10, k, 6).

The lines AB and CD intersect at the point P.

(i) Find the value of k. [6]

(ii) Find the coordinates of P. [2]

(iii) Find, in the form $ax + by + cz + d = 0$, the equation of the plane containing the points A, B, C and D. [4]

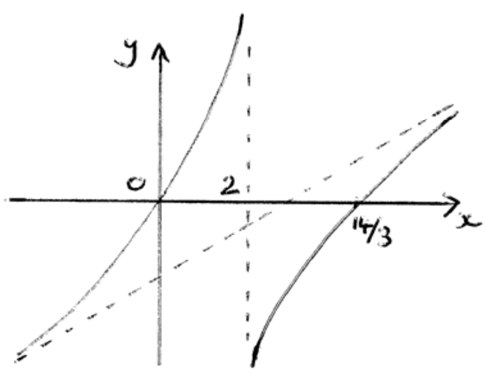
(b) The matrix $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$ defines a transformation T of the x-y plane.

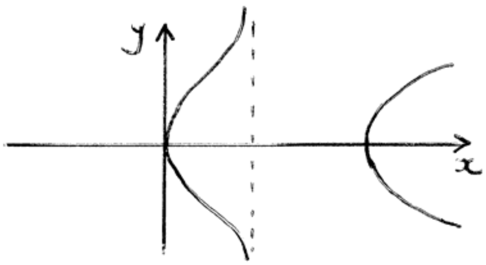
(i) Give a full geometrical description of the transformation T. [4]

(ii) The line L is transformed by T into the line $y = x - 2$.

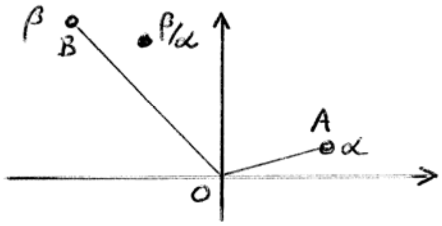
Find the equation of L. [4]

Mark Scheme 2604
June 2005

1 (i)	$x = 2$ $y = \dfrac{(x-2)(3x-8)-16}{x-2}$ $= 3x - 8 - \dfrac{16}{x-2}$ Oblique asymptote is $y = 3x - 8$	B1 M1 A1 A1 **4**	 Dividing by $(x-2)$ to obtain a linear quotient $ax + b$ with $a \neq 0$, $b \neq 0$ For quotient $3x - 8$
(ii)	$\dfrac{dy}{dx} = 3 + \dfrac{16}{(x-2)^2}$	M1 A1	Differentiation (at most one error) *or* $\dfrac{(x-2)(6x-14)-(3x^2-14x)}{(x-2)^2}$ Any correct form
	$\dfrac{dy}{dx} > 3$ so gradient is always positive	M1 A1 **4**	*or* $\dfrac{16}{(x-2)^2} > 0$ Correctly shown *These 2 marks can be earned in (iii) but only if linked to 'positive gradient'*
	OR $3x^2 - 12x + 28 = 0$ has no solutions, since $12^2 - 4 \times 3 \times 28 = -192 < 0$ M1 so $\dfrac{3x^2 - 12x + 28}{(x-2)^2} > 0$ for all x A1		*or* $x = 2 \pm 2.3\mathrm{j}$ *or* $3(x-2)^2 + 16$ Correctly shown *SR* $(3x-2)^2 + 24$ scores M1A0
(iii)		B1 B1 B1 **3**	LH section: positive gradient, through O RH section: positive gradient, through $(\frac{14}{3}, 0)$ *(Accept 4.6 to 4.7 on an accurate graph)* Fully correct shape, approaching asymptotes correctly

(iv)	$\dfrac{x(3x-14)}{x-2} = 20$ when $3x^2 - 34x + 40 = 0$ $x = \frac{4}{3},\ 10$ $\dfrac{x(3x-14)}{x-2} < 20$ when $x < \frac{4}{3},\quad 2 < x < 10$	M1 M1 A1 M1 A1A1 **6**	Obtaining quadratic equation *(condone inequality)* Solving to obtain 2 values of x or factors $(3x-4)(x-10)$ Considering intervals defined by critical values $\frac{4}{3},\ 2,\ 10$ (ft) *Condone 1.33 but not 1.3*
(v)		B1 ft B1 ft B1 **3**	No curve in 'negative regions' and curve in 'positive regions' Symmetry in x-axis Fully correct shape, including infinite gradients when crossing x-axis *(condone one 'doubtful' case)*

2 (a)	$\sum_{1}^{n}(5r^3+9r^2)$ $=\frac{5}{4}n^2(n+1)^2+\frac{9}{6}n(n+1)(2n+1)$ $=\frac{1}{4}n(n+1)(5n^2+17n+6)$ $=\frac{1}{4}n(n+1)(n+3)(5n+2)$	M1 A1A1 M1 A1 **5**	Multiplying out and using formulae
(b)	$\frac{3^{r+1}}{r+1}-\frac{3^r}{r}=\frac{3^{r+1}r-3^r(r+1)}{r(r+1)}$ $=\frac{3^r(3r-r-1)}{r(r+1)}=\frac{3^r(2r-1)}{r(r+1)}$ $\sum_{1}^{n}\frac{3^r(2r-1)}{r(r+1)}=\sum_{1}^{n}\left(\frac{3^{r+1}}{r+1}-\frac{3^r}{r}\right)$ $=\left(\frac{3^2}{2}-\frac{3^1}{1}\right)+\left(\frac{3^3}{3}-\frac{3^2}{2}\right)+\ldots+\left(\frac{3^{n+1}}{n+1}-\frac{3^n}{n}\right)$ $=\frac{3^{n+1}}{n+1}-3$	M1 A1 (ag) M1 A1 M1 A1 **6**	Three terms correct Cancelling to leave one fraction at the beginning and one fraction at the end
(c)	When $n=1$, $\text{LHS}=\frac{2-1}{1^2\times 2^2}=\frac{1}{4}$ $\text{RHS}=\frac{1^2}{2^2}=\frac{1}{4}=\text{LHS}$ Assuming it is true for $n=k$, $\sum_{1}^{k+1}=\frac{k^2}{(k+1)^2}+\frac{2(k+1)^2-1}{(k+1)^2(k+2)^2}$ $=\frac{k^2(k^2+4k+4)+2k^2+4k+1}{(k+1)^2(k+2)^2}$ $=\frac{k^4+4k^3+6k^2+4k+1}{(k+1)^2(k+2)^2}$ $=\frac{(k+1)^4}{(k+1)^2(k+2)^2}=\frac{(k+1)^2}{(k+2)^2}$ True for $n=k \Rightarrow$ True for $n=k+1$ Hence true for all positive integers n	B1 M1 A2 M1 M1 A1 A1 A1 **9**	Attempt at $S_k+(k+1)$ st term Give A1 if one slip Correctly obtained Stated or clearly implied *Dependent on previous 7 marks*

3 (a)(i)	$\lvert\alpha\rvert = \sqrt{2}$, $\arg\alpha = \frac{1}{12}\pi$ $\lvert\beta\rvert = 4\sqrt{2}$, $\arg\beta = \frac{3}{4}\pi$ $\left\lvert\frac{\beta}{\alpha}\right\rvert = 4$ $\arg\frac{\beta}{\alpha} = \frac{3}{4}\pi - \frac{1}{12}\pi = \frac{2}{3}\pi$ 	B1 B1B1 B1 ft B1 ft B1 B1 B1 **8**	*SR* Just $4(\cos\frac{2}{3}\pi + \mathrm{j}\sin\frac{2}{3}\pi)$: B1 only *For each of the following, withhold the first B1 so earned but award subsequent marks:* *Non-exact values for modulus* *Non-exact values for argument* *Arguments given in degrees* α in first quadrant β in second quadrant β/α in second quadrant with smaller argument than β *(Dependent on β in second quadrant)* Maximum B2 for diagram if points are not labelled
(ii)	$\frac{\beta}{\alpha} = 4(\cos\frac{2}{3}\pi + \mathrm{j}\sin\frac{2}{3}\pi)$ $= -2 + 2\sqrt{3}\mathrm{j}$	M1 A1 **2**	A complete exact method is required (Just $-2 + 2\sqrt{3}\mathrm{j}$ with no working scores M0)
(iii)	Line AB	B1 ft	
	$\mathrm{AB}^2 = (\sqrt{2})^2 + (4\sqrt{2})^2 - 2(\sqrt{2})(4\sqrt{2})\cos\mathrm{A\hat{O}B}$ $= 2 + 32 - 16\cos\frac{2}{3}\pi$ $= 42$ $\lvert\alpha - \beta\rvert = \sqrt{42}$	M1 A1 ft A1 A1 **5**	Use of cos rule A0 if not exact for $\mathrm{A\hat{O}B} = \frac{2}{3}\pi$

	OR $\lvert \alpha - \beta \rvert^2 = (\sqrt{2}\cos\tfrac{1}{12}\pi + 4)^2 + (\sqrt{2}\sin\tfrac{1}{12}\pi - 4)^2$ M1A1 $= 34 + 8\sqrt{2}\,(\cos\tfrac{1}{12}\pi - \sin\tfrac{1}{12}\pi)$ A1 $= 34 + 8\sqrt{2}\,\sqrt{2}\cos(\tfrac{1}{12}\pi + \tfrac{1}{4}\pi)$ $\lvert \alpha - \beta \rvert = \sqrt{42}$ A1		A0 if not exact Correct intermediate step required
(b)	Let $z = a + b\mathrm{j}$, $z^* = a - b\mathrm{j}$ $\mathrm{j}(a + b\mathrm{j}) + (2 + \mathrm{j})(a - b\mathrm{j}) = 10 - 2\mathrm{j}$ Real parts: $-b + 2a + b = 10$ Imaginary parts: $a + a - 2b = -2$ $a = 5,\ b = 6$ $z = 5 + 6\mathrm{j}$	M1 M1 A1 A1 A1 **5**	Equating real or imaginary parts *Correct answer always scores 5 marks*

4 (a)(i)	$x = 3 + 10\lambda = 1 + 9\mu$ (1) $y = 4 + 5\lambda = 2 + (k-2)\mu$ (2) $z = 7 - 5\lambda = 3 + 3\mu$ (3) Solving (1) and (3), $\lambda = \frac{2}{5}$, $\mu = \frac{2}{3}$ Substitute into (2), $4 + 2 = 2 + \frac{2}{3}(k-2)$ $k = 8$	M1 A1A1 M1 M1 A1 **6**	Equating (at least two) components *using different parameters* Two correct equations Finding λ *or* μ Equation for *k*
(ii)	$x = 7$, $y = 6$, $z = 5$ P is $(7, 6, 5)$	M1 A1 **2**	Finding x, y, z
	Alternative for (i) and (ii) Solving $\frac{x-3}{10} = \frac{z-7}{-5}$ and $\frac{x-1}{9} = \frac{z-3}{3}$ M2A1A1 $x = 7$, $z = 5$ $\frac{x-3}{10} = \frac{y-4}{5} \Rightarrow y = 6$ M1 P is $(7, 6, 5)$ A1 $\frac{x-1}{9} = \frac{y-2}{k-2} \Rightarrow \frac{7-1}{9} = \frac{6-2}{k-2}$ M1 $k = 8$ A1		For M2, must obtain a value for *x* or *z*
(iii)	Normal is $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$ Equation is $3x - 5y + z = 3 \times 3 - 5 \times 4 + 7$ $3x - 5y + z + 4 = 0$	M1A1 M1 A1 **4**	or other method for finding normal Using a point to find the constant
	OR $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ Eliminating λ and μ, M2 $3x - 5y + z + 4 = 0$ A2		Give A1 for $3x - 5y + z$
(b)(i)	Rotation Centre O $\cos\theta = 0.8$, $\sin\theta = -0.6$ Through 0.64 rad ($37°$) clockwise	M1 A1 M1 A1 **4**	either one (or $\tan\theta = -0.75$) *Allow* through $-37°$, *etc*

(ii)	Suppose $(x,\ y)$ is on L $\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.8x + 0.6y \\ -0.6x + 0.8y \end{pmatrix}$ $-0.6x + 0.8y = 0.8x + 0.6y - 2$ $y = 7x - 10$	M1 A1 M1 A1 **4**	For $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$
	OR $\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{T}^{-1}\begin{pmatrix} t \\ t-2 \end{pmatrix} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}\begin{pmatrix} t \\ t-2 \end{pmatrix}$ M1 $x = 0.2t + 1.2,\ \ y = 1.4t - 1.6$ A1 Eliminating t, M1 $y = 7x - 10$ A1		Using $\mathbf{T}^{-1}$ to transform general point on $y = x - 2$ or *two* particular points *or* images of two points correct Obtaining equation in x, y

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General Comments

There was a wide range of performance on this paper. About one quarter of the candidates scored 50 marks or more (out of 60), with many of these showing confidence and efficiency in applying the various techniques with apparent ease. On the other hand, about 20% of candidates scored fewer than 30 marks, and some of these appeared to be unfamiliar with the standard topics being examined. Some candidates made very heavy weather of the algebra in questions 2 and 3, and ran out of time, but the great majority were able to complete the paper. Quite a few answered all four questions; in almost every case the time could have been better spent concentrating on three questions and eliminating careless errors. Question 1 was attempted by almost every candidate, and question 4 was by far the least popular.

Comments on Individual Questions

1) **Curve sketching and Inequalities**

This was answered well, with half the attempts scoring 15 marks or more (out of 20).

(i) The equation of the vertical asymptote was almost invariably given correctly, but the oblique asymptote caused some difficulty. Although some gave $y = 3x - 14$ or $y = 3x$, most candidates did attempt a process of division, which was often spoilt by careless sign errors.

(ii) Candidates who started from $y = 3x - 8 - \dfrac{16}{x-2}$ were easily able to differentiate and conclude that the gradient is always positive. However, the great majority applied the quotient rule to the equation in its original form. This is possibly a safer strategy (in case errors had been made in the division), but it involved considerably more work. The gradient was usually found correctly, although errors such as $x(3x-14) = 3x^2 - 14$ and $-(3x^2 - 14x) = -3x^2 - 14x$ did occur frequently. It was then necessary to show that the quadratic expression in the numerator ($3x^2 - 12x + 28$) is always positive. Arguments such as '$3x^2 + 28$ is always greater than $12x$' were often stated but very rarely justified. It was not sufficient just to state 'there are no stationary points' without any justification. The usual approach was to show that the discriminant is negative, although many did not then state that this implies the desired result. Very few answered this by completing the square.

(iii) Much good curve sketching was seen here, although the presentation and clarity varied from excellent to very poor. It should not be necessary to use graph paper, but very many candidates chose to do so. Sketches were expected to include the asymptotes, to identify the point of intersection on the positive x-axis, and to show clearly how the curve approaches its asymptotes.

(iv) Most candidates found the critical values which give equality, but many did not know what to do next. Those who simply looked at their graph and wrote down the solution usually obtained the correct answer.

(v) The square root graph was generally well understood, but very many lost a mark for not showing clearly the infinite gradients where the curve crosses the x-axis.

2) This was the best answered question, with an average mark of about 15, and about 20% of the attempts scored full marks.

(a) **Summing a series, using standard formulae**

Most candidates obtained a correct expression for the sum of the series, but many were unable to write it as a product of linear factors.

(b) **Summing a series, using the difference method**

The identity at the beginning caused surprisingly many problems, with many writing $3r-(r+1)=3r-r+1$ despite the printed result including $2r-1$, and several tried to use partial fractions. The method of differences was well understood and was usually applied accurately, although some gave the final answer in terms of r instead of n.

(c) **Proof by induction**

This was well understood, and there were very many fully correct solutions. The main cause of loss of marks was faulty algebra.

3) **Complex numbers**

(a)(i) Finding the moduli and arguments was done well, except that the argument of β was often given as $-\frac{1}{4}\pi$ instead of $\frac{3}{4}\pi$. On the Argand diagram, α and β were usually positioned correctly, but $\dfrac{\beta}{\alpha}$ was less frequently right.

(ii) The obvious approach was to use the modulus and argument and write

$4(\cos\frac{2}{3}\pi + \mathrm{j}\sin\frac{2}{3}\pi)$; many did this and obtained the correct answer easily. However, many used much more complicated methods, such as attempting to deal with $\dfrac{-4+4\mathrm{j}}{\sqrt{2}(\cos\frac{1}{12}\pi + \mathrm{j}\sin\frac{1}{12}\pi)}$ directly.

(iii) The line was very often drawn correctly. This was intended to draw attention to the triangle and encourage the use of the cosine rule, and candidates who used this method were usually successful. However, most ignored the hint and either omitted the calculation or evaluated $\alpha - \beta$ and hence found its modulus. This often produced the correct answer of $\sqrt{42}$, but full credit was given only when a fully exact method had been shown.

(b) Most candidates knew that they should substitute $z = a + b\mathrm{j}$ into the equation, and the correct answer was very frequently obtained. The most common error was to equate the imaginary parts as $2a - 2b = 2$ instead of $2a - 2b = -2$.

4) **Vectors and Matrices**

This question was attempted by only about one third of the candidates. It was also the worst answered question, with an average mark of about 12.

(a)(i) Most candidates used the correct method of writing the equations of the two lines with different parameters, equating components, and solving the resulting simultaneous equations to find *k*. This was often carried out accurately, although arithmetic slips were very common.

(ii) Most candidates understood how to find the point of intersection.

(iii) Almost every candidate used the vector product of **AB** and **CD** to find a vector normal to the plane. This was perhaps not surprising, given the work which had already been done, but of course it gave the wrong answer when the value of *k* was incorrect. Candidates could have played it safe by using only the given points A, B and C.

(b)(i) Surprisingly many candidates were unable to describe the transformation defined by the given matrix. Even when it was recognised as a rotation, a full description (including the centre, angle and sense of rotation) was rarely given.

(ii) Correct answers to this part were quite rare. There was a lot of confusion between the object line and the image line, for example finding the image of $(x,\ x-2)$ under the given matrix instead of its inverse. Another common error was equating the image of $(x,\ y)$ to $(x,\ x-2)$ using the same symbol *x* in both the object point and the image point.